

voltage across the combined RC element that is more constant than it would have been without the presence of the C .

In a one-loop circuit, current is solenoidal, i.e., it is continuous. It is the same at every point in the loop. There are no exit or entrance points where current can enter or leave the loop. Consider an RLC single loop circuit that also contains a voltage source. The value of the current in the conductors is the same as the current value in the inductor and in the resistor. But how about between the plates of the capacitor? If we assume a perfect dielectric material between the plates, there cannot be an ohmic current in there. The problem was resolved by Maxwell by his postulating an *equivalent current* (now called the *displacement current*) given by the second term in one of his equations, namely:

$$\nabla \times \mathbf{B} = \mu \left(\mathbf{j} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \right) \quad (1)$$

where both terms inside the parenthesis have units of current density (Amps per sq. meter). The meaning of this expression is that a magnetic field (the left hand side of (1)) is created by an ohmic current density, j , flowing in the resistive path, plus an equally effective “displacement current density” (the right hand side of (1)).

Between the plates of a capacitor, C , the ohmic current density, j , has zero value, so the displacement term takes on that same value as appears in the other elements and wires in the loop.

This can be shown, from first principles as follows: A capacitor is defined as being a circuit element wherein its terminal voltage, v , is proportional to the amount of charge, q , stored in it. (q is the amount of charge added to the positive plate and simultaneously removed from the negative plate.) Thus the “size” of the capacitor, C , is determined by

$$C = \frac{q}{v}. \quad (2)$$

The larger the capacitance, the more charge it can store without changing the value of v . An analogy is a bucket for holding water. The water level is analogous to the capacitor’s voltage. A large bucket will hold more water at a given water-level, than will a smaller bucket. A bucket with vertical sides is a linear bucket. A cone-shaped Pilsner beer glass is non-linear.

Solving (2) for v and taking the derivative with respect to time, t , gives:

$$\frac{dv}{dt} = \frac{1}{C} \frac{dq}{dt} = \frac{1}{C} i \quad (3)$$

or

$$i = C \frac{dv}{dt} \quad (4)$$

This is the defining relationship for a linear capacitor in an electric circuit. [The time rate of increase in the height of the water in the bucket is proportional to the flow rate of incoming water.]

The electric field strength is defined as $E=v/d$, (The units of E are volts per meter.) E is the electrical force experienced by a unit positive charge within that field. If the area of the capacitor’s plates is A , then, by definition, the stored *charge density* is $\rho = \frac{q}{A}$ (Coulombs per square meter). Also, one of Maxwell’s equations (Gauss’ Law) says the total E -field (“flux”) emanating from the area enclosing a charge, q , is proportional to the total charge, q .

$$\epsilon EA = q \quad (5)$$

where ϵ is the constant of proportionality. So, equation (5) indicates that, inside a capacitor holding a fixed charge, q , a higher value of ϵ will result in a lower value of E , the force experienced by a charge between the plates and vice-versa. It follows from (5) and the definition of charge density, ρ , that

$$E = \frac{1}{\epsilon} \frac{q}{A} = \frac{\rho}{\epsilon} \quad (6)$$

Therefore from expressions (2) and (6):

$$C = \frac{q}{v} = \frac{q}{Ed} = \frac{q\epsilon}{\rho d} = \frac{qA\epsilon}{qd} = \frac{A\epsilon}{d} \quad (7)$$

Expression (7) is the definition of the capacitance value of a parallel plate capacitor. Because in such a parallel plate capacitor $E = v/d$, where d is the plate separation distance and the terminal current is $C dv/dt$ (see expression (4)), it follows that this total capacitor (“displacement”) current is the displacement current *density* multiplied by the plate area, A . Therefore we have

$$i = C \frac{dV}{dt} = \frac{A\epsilon}{d} \frac{dV}{dt} = A\epsilon \frac{dE}{dt} \quad (8)$$

so the displacement current density is

$$j = \frac{i}{A} = \epsilon \frac{dE}{dt} \quad (9)$$

This is the term Maxwell added to his equation (see expression (1) above

This complies with the requirement that the current be the same everywhere in the loop (including between the capacitor plates). Expression (9) makes the supremely important statement that ***a time-varying electric field is just as effective in creating a magnetic field as a normal ohmic current, j.***

There is no limitation placed on what the maximum distance, d , between the capacitor plates can be. Therefore, thinking about some very large region in cosmic space wherein there is no ohmic current density, j , nor Birkeland Currents to carry one, a magnetic field can be created there by a ***time-varying electric field, E(t).***

Ohmic currents require conducting paths. In the cosmos Birkeland currents can serve that purpose. But even if no such conducting (plasma) pathways are present in some region of space, magnetic fields may be produced there by time-varying electric fields which do not require any such pathways. Thus Maxwell’s “displacement current” should join the astrophysicists’ favorite “dynamo effect” (another word for induction) as a possible cause of the creation of magnetic fields and, thereby, other electric currents.

Maxwell died in November of 1879, so this mechanism has been known for about 140 years – but is not yet in the lexicon of astrophysicists. It ought to be.

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ⁱ Solar polar plumes lecture: <https://www.youtube.com/watch?v=HiW9eZhCt0g>